Schwarzschild-Milne eq.

update: 2017 Nov. 29, author: Sho K. NAKAMURA

[?] F_ν

$$
\begin{aligned}
F_\nu(\tau_\nu) &= 2\pi\int_{-1}^{1} I_\nu \mu d\mu = 2\pi\left(\int_0^1 I_\nu^+ \mu d\mu + \int_{-1}^{0} I_\nu^- \mu d\mu\right) \\
&= 2\pi\left(\int_0^1 \mu d\mu \int_{\tau_\nu}^{\infty} dt \frac{S_\nu(t)}{\mu} e^{-(t-\tau_\nu)/\mu} + \int_{-1}^{0} \mu d\mu \int_0^{\tau_\nu} dt \frac{S_\nu(t)}{-\mu} e^{-(\tau_\nu - t)/(-\mu)}\right) \\
&= 2\pi\left(\int_{\tau_\nu}^{\infty} dt S_\nu(t) \int_0^1 d\mu e^{-(t-\tau_\nu)/\mu} - \int_0^{\tau_\nu} dt S_\nu(t) \int_{-1}^{0} d\mu e^{-(\tau_\nu - t)/(-\mu)}\right) \qquad (1)
\end{aligned}
$$

第一項で $\omega = 1/\mu$ と変数変換すると、$d\mu = -d\omega/\omega^2$ より

$$
\int_0^1 d\mu e^{-(t-\tau_\nu)/\mu} = \int_1^{\infty} d\omega \frac{e^{-(t-\tau_\nu)\omega}}{\omega^2} = E_2(t-\tau_\nu) \qquad (2)
$$

第二項で $\omega = -1/\mu$ と変数変換すると、$d\mu = d\omega/\omega^2$ より

$$
\int_{-1}^{0} d\mu e^{-(\tau_\nu - t)/(-\mu)} = \int_1^{\infty} d\omega \frac{e^{-(\tau_\nu - t)\omega}}{\omega^2} = E_2(\tau_\nu - t) \qquad (3)
$$

$$
\therefore\ F_\nu(\tau_\nu) = 2\pi\left(\int_{\tau_\nu}^{\infty} dt S_\nu(t) E_2(t-\tau_\nu) - \int_0^{\tau_\nu} dt S_\nu(t) E_2(\tau_\nu - t)\right) \qquad (4)
$$

[?] K_ν

$$
\begin{aligned}
K_\nu(\tau_\nu) &= \frac{1}{4\pi}\int_{-1}^{1} I_\nu \mu^2 d\mu = \frac{1}{2}\left(\int_0^1 I_\nu^+ \mu^2 d\mu + \int_{-1}^{0} I_\nu^- \mu^2 d\mu\right) \\
&= \frac{1}{2}\left(\int_0^1 \mu^2 d\mu \int_{\tau_\nu}^{\infty} dt \frac{S_\nu(t)}{\mu} e^{-(t-\tau_\nu)/\mu} + \int_{-1}^{0} \mu^2 d\mu \int_0^{\tau_\nu} dt \frac{S_\nu(t)}{-\mu} e^{-(\tau_\nu - t)/(-\mu)}\right) \\
&= \frac{1}{2}\left(\int_{\tau_\nu}^{\infty} dt S_\nu(t) \int_0^1 d\mu \mu e^{-(t-\tau_\nu)/\mu} - \int_0^{\tau_\nu} dt S_\nu(t) \int_{-1}^{0} d\mu \mu e^{-(\tau_\nu - t)/(-\mu)}\right) \qquad (5)
\end{aligned}
$$

第一項で $\omega = 1/\mu$ と変数変換すると、$d\mu = -d\omega/\omega^2$ より

$$
\int_0^1 d\mu \mu e^{-(t-\tau_\nu)/\mu} = \int_1^{\infty} d\omega \frac{e^{-(t-\tau_\nu)\omega}}{\omega^3} = E_3(t-\tau_\nu) \qquad (6)
$$

第二項で $\omega = -1/\mu$ と変数変換すると、$d\mu = d\omega/\omega^2$ より

$$
-\int_{-1}^{0} d\mu \mu e^{-(\tau_\nu - t)/(-\mu)} = \int_1^{\infty} d\omega \frac{e^{-(\tau_\nu - t)\omega}}{\omega^3} = E_3(\tau_\nu - t) \qquad (7)
$$

$$
\therefore\ K_\nu(\tau_\nu) = \frac{1}{2}\int_0^{\infty} dt S_\nu(t) E_3(|t-\tau_\nu|) \qquad (8)
$$