General Relativity

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[?] テンソルの共変微分

$$\nabla_\mu A^\alpha = \partial_\mu A^\alpha + A^\beta \Gamma^\alpha_{\beta\mu} \tag{1}$$

$$\nabla_\mu B_\alpha = \partial_\mu B_\alpha - B_\beta \Gamma^\beta_{\mu\alpha} \tag{2}$$

より

$$\begin{aligned} \nabla_\mu T_{\alpha\beta} &= \nabla_\mu (A_\alpha B_\beta) = (\nabla_\mu A_\alpha) B_\beta + A_\alpha (\nabla_\mu B_\beta) = (\partial_\mu A_\alpha) B_\beta - A_\nu \Gamma^\nu_{\mu\alpha} B_\beta + A_\alpha (\partial_\mu B_\beta) - A_\alpha B_\nu \Gamma^\nu_{\mu\beta} \\ &= \partial_\mu (A_\alpha B_\beta) - \Gamma^\nu_{\mu\alpha} A_\nu B_\beta - \Gamma^\nu_{\mu\beta} A_\alpha B_\nu \implies \therefore \nabla_\mu T_{\alpha\beta} = \partial_\mu T_{\alpha\beta} - \Gamma^\nu_{\mu\alpha} T_{\nu\beta} - \Gamma^\nu_{\mu\beta} T_{\alpha\nu} \end{aligned} \tag{3}$$

[?] 計量テンソルの共変微分

$$\begin{aligned} \nabla_\mu g_{\alpha\beta} &= \partial_\mu g_{\alpha\beta} - \Gamma^\nu_{\mu\alpha} g_{\nu\beta} - \Gamma^\nu_{\mu\beta} g_{\alpha\nu} \\ &= \partial_\mu g_{\alpha\beta} - \frac{1}{2} g^{\nu\gamma} (g_{\beta\mu,\alpha} + g_{\beta\alpha,\mu} - g_{\mu\alpha,\beta}) g_{\nu\beta} - \frac{1}{2} g^{\nu\gamma} (g_{\gamma\mu,\beta} + g_{\gamma\beta,\mu} - g_{\mu\beta,\alpha}) g_{\alpha\nu} \\ &= \partial_\mu g_{\alpha\beta} - \frac{1}{2} (g_{\beta\mu,\alpha} + g_{\beta\alpha,\mu} - g_{\mu\alpha,\beta}) \delta^\gamma_\beta - \frac{1}{2} (g_{\gamma\mu,\beta} + g_{\gamma\beta,\mu} - g_{\mu\beta,\alpha}) \delta^\gamma_\alpha = \partial_\mu g_{\alpha\beta} - \frac{1}{2} 2 g_{\alpha\beta,\mu} = 0 \end{aligned} \tag{4}$$