Induction equation

date: 2017 Sept. 10, author: Sho K. NAKAMURA

Induction eq. in Cylindrical and Spherical geometry.

誘導方程式

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \tag{1}$$

をシミュレーションしやすいように発散の形に書き換える。

円筒座標系)

$$\begin{aligned}\frac{\partial B_R}{\partial t} &= \frac{1}{R}\frac{\partial}{\partial \varphi}(v_R B_\varphi - B_R v_\varphi) + \frac{\partial}{\partial z}(v_R B_z - B_R v_z) \\ &= \frac{1}{R}\frac{\partial}{\partial R}\{R(v_R B_R - B_R v_R)\} + \frac{1}{R}\frac{\partial}{\partial \varphi}(v_R B_\varphi - B_R v_\varphi) + \frac{\partial}{\partial z}(v_R B_z - B_R v_z) \\ &\Longrightarrow \frac{\partial B_R}{\partial t} + \nabla \cdot (B_R \mathbf{v} - v_R \mathbf{B}) = 0\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial B_\varphi}{\partial t} &= \frac{\partial}{\partial z}(v_\varphi B_z - B_\varphi v_z) + \frac{\partial}{\partial R}(v_\varphi B_R - B_\varphi v_R) \\ &= \frac{1}{R}\frac{\partial}{\partial R}\{R(v_\varphi B_R - B_\varphi v_R)\} + \frac{1}{R}\frac{\partial}{\partial \varphi}(v_\varphi B_\varphi - B_\varphi v_\varphi) + \frac{\partial}{\partial z}(v_\varphi B_z - B_\varphi v_z) - \frac{v_\varphi B_R - B_\varphi v_R}{R} \\ &\Longrightarrow \frac{\partial B_\varphi}{\partial t} + \nabla \cdot (B_\varphi \mathbf{v} - v_\varphi \mathbf{B}) = \frac{B_\varphi v_R - v_\varphi B_R}{R}\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial B_z}{\partial t} &= \frac{1}{R}\frac{\partial}{\partial R}\{R(v_z B_R - B_z v_R)\} + \frac{1}{R}\frac{\partial}{\partial \varphi}(v_z B_\varphi - B_z v_\varphi) + \frac{\partial}{\partial z}(v_z B_z - B_z v_z) \\ &\Longrightarrow \frac{\partial B_z}{\partial t} + \nabla \cdot (B_z \mathbf{v} - v_z \mathbf{B}) = 0\end{aligned} \tag{4}$$

3次元極座標)

$$\begin{aligned}\frac{\partial B_r}{\partial t} &= \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\{\sin\theta(v_r B_\theta - B_r v_\theta)\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(v_r B_\varphi - B_r v_\varphi) \\ &= \frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2(v_r B_r - B_r v_r)\right\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\{\sin\theta(v_r B_\theta - B_r v_\theta)\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(v_r B_\varphi - B_r v_\varphi) \\ &\Longrightarrow \frac{\partial B_R}{\partial t} + \nabla \cdot (B_r \mathbf{v} - v_r \mathbf{B}) = 0\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial B_\theta}{\partial t} &= \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(v_\theta B_\varphi - B_\theta v_\varphi) + \frac{1}{r}\frac{\partial}{\partial r}\{r(v_\theta B_r - B_\theta v_r)\} \\ &= \frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2(v_\theta B_r - B_\theta v_r)\right\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\{\sin\theta(v_\theta B_\theta - B_\theta v_\theta)\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(v_\theta B_\varphi - B_\theta v_\varphi) - \frac{v_\theta B_r - B_\theta v_r}{r} \\ &\Longrightarrow \frac{\partial B_\theta}{\partial t} + \nabla \cdot (B_\theta \mathbf{v} - v_\theta \mathbf{B}) = \frac{B_\theta v_r - v_\theta B_r}{r}\end{aligned} \tag{6}$$

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\begin{aligned}
\frac{\partial B_\theta}{\partial t} &= \frac{1}{r}\frac{\partial}{\partial r}\left\{r(v_\varphi B_r - B_\varphi v_r)\right\} + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\varphi B_\theta - B_\varphi v_\theta) \\
&= \frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2(v_\varphi B_r - B_\varphi v_r)\right\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left\{\sin\theta(v_\varphi B_\theta - B_\varphi v_\theta)\right\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(v_\varphi B_\varphi - B_\varphi v_\varphi) \\
&\quad - \frac{v_\varphi B_r - B_\varphi v_r}{r} - \frac{\cot\theta}{r}(v_\varphi B_\theta - B_\varphi v_\theta) \\
\Longrightarrow &\quad \frac{\partial B_\theta}{\partial t} + \nabla\cdot(B_\varphi \mathbf{v} - v_\varphi \mathbf{B}) = \frac{B_\varphi v_r - v_\varphi B_r}{r} + \frac{\cot\theta}{r}(B_\varphi v_\theta - v_\varphi B_\theta) \qquad (7)
\end{aligned}
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円筒座標・極座標系では曲率による源泉項が入る。